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$\mathcal{N} = 1/2$ quiver gauge theories from open strings with R-R fluxes*

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Abstract: We consider a four dimensional $\mathcal{N} = 1$ gauge theory with bifundamental matter and a superpotential, defined on stacks of fractional branes. By turning on a flux for the R-R graviphoton field strength and computing open string amplitudes with insertions of R-R closed string vertices, we introduce a non-anticommutative deformation and obtain the $\mathcal{N} = 1/2$ version of the theory. We also comment on the appearance of a new structure in the effective lagrangian.

Keywords: Gauge Symmetry, D-branes, Extended Supersymmetry.

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JHEP05(2005)047

Contents

1. Introduction

The study of deformed gauge theories has recently attracted a lot of interest, especially after it has become evident the connection between field theory deformation parameters and non-trivial geometric backgrounds. This connection is particularly clear in a string theory context where the gauge theories describe the low-energy dynamics of open strings attached to D-branes and the deformation parameters are associated to non-trivial fluxes for some closed string fields to which the D-branes can couple. The most notable example of this relation is provided by the non-commutative gauge theories which can be efficiently described in terms of open strings propagating in a background with a constant NS-NS $B_{\mu\nu}$ field [1]. More recently, other types of backgrounds have been considered by turning on fluxes for suitable combinations of the anti-symmetric tensor fields of the closed string spectrum. Among the various possibilities that have been explored, there is the one in which a graviphoton background of the R-R sector is turned on.

As explained in refs. [2 − 6], a constant self-dual graviphoton field strength $C_{\mu\nu}$ induces a deformation of the four dimensional superspace [7, 8] in which the fermionic coordinates cease to be anticommuting Grassmann variables and become elements of a Clifford algebra, namely

$$\{\theta^\alpha, \theta^\beta\} = C^{\alpha\beta}\,, \qquad \{\theta^\alpha, \bar\theta^{\dot\beta}\} = \{\bar\theta^{\dot\alpha}, \bar\theta^{\dot\beta}\} = 0 \tag{1.1}$$

where $C^{\alpha\beta} = \frac{1}{4}C_{\mu\nu}(\sigma^{\mu\nu})^{\alpha\beta}$. The non-vanishing anticommutator in (1.1) breaks the four dimensional Lorentz group $\mathrm{SU}(2)_\mathrm{L}\times\mathrm{SU}(2)_\mathrm{R}$ to $\mathrm{SU}(2)_\mathrm{L}$ and reduces the number of conserved supercharges by a factor of two. Therefore, a graviphoton background deforms a $\mathcal{N}=1$ field theory in four dimensions into a $\mathcal{N}=1/2$ theory with only two supercharges. Furthermore, new types of interactions and couplings are induced by the non-anticommutative structure of superspace.

Supersymmetric field theories based on non-anticommutative superspaces (which we will call simply non-anticommutative, or NAC, field theories) have been the subject of vast investigation in the recent past from many different points of view [9 − 24]. In this paper, extending our previous work [25], we will analyze $\mathcal{N}=1/2$ gauge theories with matter in the fundamental or bifundamental representation working explicitly in a stringy set-up. In particular, we will engineer a $\mathcal{N}=1$ gauge theory in four dimensions by considering stacks of fractional D3-branes in the orbifold $\mathbb{C}^3/(\mathbb{Z}_2\times\mathbb{Z}_2)$; the open strings starting and ending on the same type of fractional D-branes describe the gauge multiplets, while the strings stretching between two different types of D-branes describe chiral and anti-chiral matter multiplets in bifundamental representations. We then demonstrate that a NAC deformation of this quiver gauge theory, including its superpotential, appears by turning on a graviphoton background with constant field strength in the R-R sector. The presence of a non trivial R-R flux modifies the dynamics of the open strings and introduces new couplings that correspond to mixed open/closed string amplitudes which we explicitly compute. These new interactions are the same as those which can be derived from the NAC deformation of superspace. However, we also find an extra coupling which cannot be immediately obtained from the NAC superspace. Our approach provides in principle a unified way of treating various deformations on gauge theories by computing mixed open/closed string amplitudes, and shows that, at least when the flux is constant, the NSR formulation of string theory allows to treat also a R-R background.

This paper is organized as follows: in section 2, using a superspace approach, we first discuss the NAC deformation of a $\mathrm{U}(N)$ gauge theory with fundamental matter, and then the NAC structure of a quiver gauge theory with group $\mathrm{U}(N_0)\times\mathrm{U}(N_1)\times\mathrm{U}(N_2)\times\mathrm{U}(N_3)$ and matter in bifundamental representations. In section 3 we show how to engineer the above quiver theory, including its superpotential, with fractional D3 branes of type-IIB string theory in the orbifold $\mathbb{C}^3/(\mathbb{Z}_2\times\mathbb{Z}_2)$, while in section 4 we explicitly derive the deformation induced by a R-R graviphoton background on the massless open string dynamics by computing mixed open/closed string amplitudes in the NSR formalism. Finally, in appendix A we list our conventions and collect some technical details that are useful to reproduce our calculations.

JHEP05(2005)047

2. $\mathcal{N}=1/2$ gauge theories with fundamental matter

In this section we review the NAC deformation of $\mathcal{N}=1$ gauge theories; we use a superspace approach first to describe deformed superfields, and then to introduce gauge invariant actions for NAC theories with chiral matter in fundamental or bifundamental representations.

2.1 Superfields in NAC superspace

In terms of the commuting chiral coordinates $y^\mu \equiv x^\mu + \mathrm{i}\,\theta\sigma^\mu\bar\theta$, a vector superfield V in the WZ gauge has the following expansion[1]

$$V = -2\,\theta\sigma^\mu\bar\theta\,A_\mu(y) - 2\mathrm{i}\,\bar\theta\bar\theta\,\theta\lambda(y) - 2\mathrm{i}\,\theta\theta\,\bar\theta\,\overline{\lambda}(y) + \theta\theta\,\bar\theta\bar\theta\,\big(\mathrm{i}\,\partial\cdot A(y) + D(y)\big) \tag{2.1}$$

where A_μ is the gauge vector field, λ and $\overline{\lambda}$ are the gauginos and D is an auxiliary field. Clearly these components transform in the adjoint representation of the gauge group, and the residual transformations which preserve the WZ gauge are of the form

$$\mathrm{e}^V \rightarrow \mathrm{e}^{V'} = \mathrm{e}^{-\mathrm{i}\,\overline{\Xi}}\,\mathrm{e}^V\,\mathrm{e}^{\mathrm{i}\,\Xi} \tag{2.2}$$

with Ξ and $\overline{\Xi}$ given by

$$\Xi = \varepsilon(y)\,, \qquad \overline{\Xi} = \varepsilon(y) - 2\mathrm{i}\,\theta\sigma^\mu\bar\theta\,\partial_\mu\varepsilon(y) - \theta\theta\,\bar\theta\bar\theta\,\partial^2\varepsilon(y) \tag{2.3}$$

in terms of the gauge parameter $\varepsilon(y)$. Indeed, by expanding (2.2) in θ and $\bar\theta$, one can easily find the standard infinitesimal gauge transformations for the components

$$\begin{aligned} \delta A_\mu &= \partial_\mu\varepsilon + \mathrm{i}\,[A_\mu\,,\,\varepsilon]\,, \qquad & \delta\lambda &= \mathrm{i}\,[\lambda\,,\,\varepsilon] \\ \delta\overline{\lambda} &= \mathrm{i}\,\big[\overline{\lambda}\,,\,\varepsilon\big]\,, & \delta D &= \mathrm{i}\,[D\,,\,\varepsilon]\,. \end{aligned} \tag{2.4}$$

On the other hand, a chiral superfield Φ has the following θ-expansion

$$\Phi = \varphi(y) + \sqrt{2}\,\theta\chi(y) + \theta\theta\,F(y) \tag{2.5}$$

where φ is a complex scalar field, χ is a chiralino and F is an auxiliary field. Correspondingly, an anti-chiral superfield $\overline{\Phi}$ is given by

$$\overline{\Phi} = \overline{\varphi}(y) - 2\mathrm{i}\,\theta\sigma^\mu\bar\theta\,\partial_\mu\overline{\varphi}(y) - \theta\theta\,\bar\theta\bar\theta\,\partial^2\overline{\varphi}(y) + \sqrt{2}\,\bar\theta\overline{\chi}(y) - 2\sqrt{2}\mathrm{i}\,\theta\sigma^\mu\bar\theta\,\bar\theta\partial_\mu\overline{\chi}(y) + \bar\theta\bar\theta\,\overline{F}(y) \tag{2.6}$$

in terms of the conjugate components. When these fields are in the fundamental and anti-fundamental representations of the gauge group, their gauge transformations are

$$\Phi \;\rightarrow \Phi' = \mathrm{e}^{-\mathrm{i}\,\Xi}\,\Phi\,, \qquad \overline{\Phi} \rightarrow \;\overline{\Phi}' = \overline{\Phi}\,\mathrm{e}^{\mathrm{i}\,\overline{\Xi}}\,, \tag{2.7}$$

which imply the following infinitesimal transformations for the components

$$\begin{aligned} \delta\varphi &= -\mathrm{i}\,\varepsilon\,\varphi\,, \qquad & \delta\chi &= -\mathrm{i}\,\varepsilon\,\chi\,, \qquad & \delta F &= -\mathrm{i}\,\varepsilon\,F\,, \\ \delta\overline{\varphi} &= \mathrm{i}\,\overline{\varphi}\,\varepsilon\,, & \delta\overline{\chi} &= \mathrm{i}\,\overline{\chi}\,\varepsilon\,, & \delta\overline{F} &= \mathrm{i}\,\overline{F}\,\varepsilon\,. \end{aligned} \tag{2.8}$$

[1] Our euclidean conventions are given in appendix A.1.

JHEP05(2005)047

In ref. [5] the consequences of the NAC deformation (1.1) of the superspace have been analyzed and interpreted. First of all, in the presence of $C^{\alpha\beta}$ a new product among superfields, called $\star$-product, must be introduced according to

$$\Psi_1 \star \Psi_2 = \Psi_1 \exp\left(-\frac{C^{\alpha\beta}}{2}\frac{\overleftarrow{\partial}}{\partial\theta^\alpha}\frac{\overrightarrow{\partial}}{\partial\theta^\beta}\right)\Psi_2 \tag{2.9}$$

$$= \Psi_1\,\Psi_2 - C^{\alpha\beta}\left(\psi_{1\alpha} + \sqrt{2}\,\theta_\alpha f_1\right)\left(\psi_{2\beta} + \sqrt{2}\,\theta_\beta f_2\right) - \det C\, f_1\, f_2 \tag{2.10}$$

where Ψ_1 and Ψ_2 are two arbitrary superfields, ψ_α and f are, respectively, their θ^α and the $\theta\theta$ components (which in general can be functions both of y and of $\bar\theta$), and $\det C = \frac{1}{2}C^{\alpha\beta}C_{\alpha\beta} = \frac{1}{4}C^{\mu\nu}C_{\mu\nu}$. Then, the parameterization (2.1) of the vector superfield V must be modified by shifting the gaugino λ_α according to

$$\lambda_\alpha \to \lambda_\alpha - \frac{1}{2}\,C_\alpha{}^\beta\sigma^\mu_{\beta\dot\alpha}\left\{\overline{\lambda}^{\dot\alpha}\,,\,A_\mu\right\} \tag{2.11}$$

in such a way that the *standard* gauge transformations (2.4) can be derived from the $\star$-product version of (2.2), i.e.

$$\mathrm{e}^V \to \mathrm{e}^{V'} = \mathrm{e}^{-\mathrm{i}\,\overline{\Xi}} \star \mathrm{e}^V \star \mathrm{e}^{\mathrm{i}\,\Xi}\,. \tag{2.12}$$

In these expressions the exponentials are defined with the $\star$-product and the gauge parameters are given by

$$\Xi = \varepsilon(y) \tag{2.13}$$

$$\overline{\Xi} = \varepsilon(y) - 2\mathrm{i}\,\theta\sigma^\mu\bar\theta\,\partial_\mu\varepsilon(y) - \theta\theta\,\bar\theta\bar\theta\,\partial^2\varepsilon(y) + \mathrm{i}\,\bar\theta\bar\theta\,C^{\mu\nu}\,\left\{\partial_\mu\varepsilon(y)\,,\,A_\nu\right\}\,. \tag{2.14}$$

Note the appearance in $\overline{\Xi}$ of a C-dependent term that involves also the gauge field. Furthermore, from the deformed vector superfield one can obtain a deformed field strength superfield $\mathcal{W}_\alpha$ by replacing ordinary products with $\star$-products in the usual definition [5], i.e.

$$\mathcal{W}_\alpha = -\frac{1}{8}\,\overline{D}^2 \star \mathrm{e}^{-V} \star D_\alpha \star \mathrm{e}^V \tag{2.15}$$

where D_α and $\overline{D}_{\dot\alpha}$ are the standard covariant derivatives. In this way one finds that $\mathcal{W}_\alpha$ acquires a deformation term proportional to $C_\alpha{}^\beta\,\theta_\beta\,\overline{\lambda}\,\overline{\lambda}$.

This reasoning can be extended also to chiral and anti-chiral superfields (see for example ref. [11]). If one requires that the *standard* gauge transformations of the matter fields (2.8) follow from the $\star$-product version of (2.7), i.e. from

$$\Phi \to \Phi' = \mathrm{e}^{-\mathrm{i}\,\Xi} \star \Phi\,, \qquad \overline{\Phi} \to \overline{\Phi}' = \overline{\Phi} \star \mathrm{e}^{\mathrm{i}\,\overline{\Xi}}\,, \tag{2.16}$$

then the usual expansion (2.5) of the chiral superfield can be kept, but the parameterization of $\overline{\Phi}$ must be changed by replacing in (2.6) the auxiliary field according to

$$\overline{F} \to \overline{F} + 2\mathrm{i}\,C^{\mu\nu}\,\partial_\mu\big(\overline{\varphi}\,A_\nu\big) - C^{\mu\nu}\,\overline{\varphi}\,A_\mu\,A_\nu + \mathrm{i}\,a\,C^{\mu\nu}\,\overline{\varphi}\,F_{\mu\nu} + b\,\det C\,\overline{\varphi}\,\overline{\lambda}\,\overline{\lambda} \tag{2.17}$$

where a and b are free parameters. In ref. [11] the minimal choice $a = b = 0$ was made but other choices are equally acceptable. In any case, it is interesting to note that the C-deformation of superspace induces in the anti-chiral superfield $\overline{\Phi}$ the appearance of terms that depend on the gauge vector A_μ and possibly also on the gaugino $\overline{\lambda}$.

2.2 U(N) gauge theories with matter in NAC superspace

After these preliminaries, it is quite easy to write gauge invariant actions for NAC theories with chiral matter in the fundamental representation.

Let us first consider the simplest example of a theory with gauge group U(N). In this case the pure Yang-Mills part of the lagrangian is given by

$$\mathcal{L}_{\text{gauge}} = \frac{\mathrm{i}}{8\pi}\left[\int d^2\theta\tau\,\mathrm{Tr}\Big(\mathcal{W}\star\mathcal{W}\Big) - \int d^2\bar\theta\bar\tau\,\mathrm{Tr}\Big(\overline{\mathcal{W}}\star\overline{\mathcal{W}}\Big)\right] \tag{2.18}$$

where $\tau = \frac{\theta_{\text{YM}}}{2\pi} + \mathrm{i}\,\frac{4\pi}{g^2}$ is the complexified Yang-Mills coupling, and $\mathcal{W}$ is the deformed field strength superfield (2.15). Expanding (2.18) in components, we find

$$\begin{aligned}\mathcal{L}_{\text{gauge}} = \frac{1}{g^2}\,\mathrm{Tr}\Big\{&\frac{1}{2}\,F^2_{\mu\nu} - 2\mathrm{i}\,\bar\lambda\,\bar\sigma^\mu D_\mu\lambda - D^2 + 2\mathrm{i}\,C^{\mu\nu}\,F_{\mu\nu}\,\overline{\lambda}\,\overline{\lambda}\\ &- 4\,\det C\,\big(\overline{\lambda}\,\overline{\lambda}\big)^2\Big\} - \frac{\mathrm{i}\,\theta_{\text{YM}}}{32\pi^2}\,\varepsilon^{\mu\nu\rho\sigma}\,\mathrm{Tr}\,F_{\mu\nu}\,F_{\rho\sigma}\end{aligned} \tag{2.19}$$

Note that the NAC deformation does not affect the θ_{YM}-term which remains purely topological. The action (2.19), which was first written in ref. [5], can also be obtained by computing scattering amplitudes of open strings in a R-R graviphoton background as shown in ref. [25].

The matter part of the lagrangian is given by the usual expression in which ordinary products are replaced by $\star$-products, namely

$$\mathcal{L}_{\text{matt}} = \int d^2\theta\,d^2\bar\theta\Big(\overline{\Phi}\star\mathrm{e}^V\star\Phi\Big)\,. \tag{2.20}$$

The gauge invariance of $\mathcal{L}_{\text{matt}}$ is manifest from the transformation properties (2.12) and (2.16), and its explicit component form can be obtained with a straightforward calculation that leads, modulo total derivative terms, to

$$\begin{aligned}\mathcal{L}_{\text{matt}} = &\,D^\mu\overline{\varphi}\,D_\mu\varphi - \mathrm{i}\,\overline{\chi}\,\bar\sigma^\mu D_\mu\chi + \overline{F}\,F + \overline{\varphi}\,D\,\varphi + \sqrt{2}\,\mathrm{i}\,\big(\overline{\chi}\,\overline{\lambda}\,\varphi + \overline{\varphi}\,\lambda\,\chi\big) +\\ &+\sqrt{2}\,C^{\mu\nu}\,D_\mu\overline{\varphi}\,\overline{\lambda}\,\bar\sigma_\nu\,\chi + \mathrm{i}a'\,C^{\mu\nu}\,\overline{\varphi}\,F_{\mu\nu}\,F + b'\det C\;\overline{\varphi}\,\overline{\lambda}\,\overline{\lambda}\,F\end{aligned} \tag{2.21}$$

where $a' = a+1$ and $b' = b-1$ in terms of the parameters appearing in (2.17). This lagrangian was first introduced and analyzed in ref. [11] where, however, different conventions were used and the choice $a' = -b' = 1$ was made.

It can be shown in full generality that the complete system ($\mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{matt}}$) is invariant, up to total derivatives, only under a half of the original $\mathcal{N}=1$ supersymmetry transformations, namely under

$$\begin{aligned}\delta A_\mu &= \mathrm{i}\,\xi\,\sigma_\mu\,\overline{\lambda}\,, \qquad \delta D = \xi\,\sigma^\mu D_\mu\overline{\lambda}\,, \qquad \delta\overline{\lambda} = 0\,,\\ \delta\lambda &= \mathrm{i}\,\xi\,D - \frac{1}{2}\,\xi\,\sigma^{\mu\nu}\Big(F_{\mu\nu} + \mathrm{i}\,C_{\mu\nu}\,\overline{\lambda}\,\overline{\lambda} + \mathrm{i}\,\frac{g^2}{2}\,C_{\mu\nu}\,F\,\overline{\varphi}\Big)\,,\\ \delta\varphi &= \sqrt{2}\,\xi\,\chi\,, \qquad \delta\overline{\varphi} = 0\,, \qquad \delta\chi = \sqrt{2}\,\xi\,F\,,\end{aligned}$$

$$
\begin{aligned}
\delta\overline{\chi} &= -\sqrt{2}\mathrm{i}\, D_\mu\overline{\varphi}\,\xi\,\sigma^\mu\,, \qquad \delta F = 0\,,\\
\delta\overline{F} &= \sqrt{2}\mathrm{i}\,\xi\,\sigma^\mu\, D_\mu\overline{\chi} - 2\mathrm{i}\,\overline{\varphi}\,\xi\,\lambda + \\
&\quad + C^{\mu\nu}\left(2\,D_\mu\overline{\varphi}\,\xi\,\sigma_\nu\,\overline{\lambda} + (2a'-1)\,\overline{\varphi}\,\xi\,\sigma_\nu\,D_\mu\overline{\lambda} - \frac{\sqrt{2}}{4}g^2\,\overline{\varphi}\,\xi\,\sigma_{\mu\nu}\chi\,\overline{\varphi}\right)
\end{aligned}
\tag{2.22}
$$

where ξ is the chiral anti-commuting parameter. Notice the presence of C-dependent terms proportional to the coupling constant g^2 in the transformation laws of the gaugino λ and the auxiliary field $\overline{F}$, which were not previously considered. The remaining supersymmetries, associated to the anti-chiral parameter $\overline{\xi}$, are explicitly broken by the NAC deformation.

The theory described by (2.21) can be regarded as the gauged version of the $\mathcal{N} = 1/2$ Wess-Zumino model whose renormalization properties have been recently studied in the literature (see for example refs. [12]). Due to the charge carried by the chiral superfield, there is no room in (2.21) for a superpotential term, and so if we want to investigate superpotentials we have to consider a suitable extension of this theory, which we will do in the next subsection. However, it is interesting to observe that in the present context it is possible to introduce a supersymmetric and gauge invariant interaction term of the form

$$
\mathcal{L}_{\mathrm{int}} = c'\,g^2\,\det C\,(\overline{\varphi}\,F)^2 \tag{2.23}
$$

where c' is a free parameter.[2] Such a term is compatible with the $\star$-product structure of the model since it can be generated by adding in (2.17) a further shift for the auxiliary field $\overline{F}$ that respects all requirements, i.e.

$$
\overline{F} \;\rightarrow\; \overline{F} + c'\,g^2\,\det C\,\overline{\varphi}\,F\,\overline{\varphi}\,. \tag{2.24}
$$

The interaction (2.23), which survives also in the ungauged theory, is not usually included in the lagrangian of the $\mathcal{N} = 1/2$ Wess-Zumino model, since in this case, using the equation of motion for the auxiliary field, it becomes proportional to $\det C\,F^3$, i.e. to a term of the deformed Wess-Zumino superpotential. However, in ref. [12] it has been shown that a term precisely like (2.23) appears in the 1-loop divergences of the $\mathcal{N} = 1/2$ Wess-Zumino model. In section 4 we will show that an interaction of the form (2.23) naturally appears in the string realization of the NAC theories provided by D3 branes in a R-R graviphoton background.

2.3 Quiver gauge theories in NAC superspace

We now generalize the above NAC construction to the $\mathcal{N} = 1$ quiver theory with gauge group $\mathrm{U}(N_0)\times\mathrm{U}(N_1)\times\mathrm{U}(N_2)\times\mathrm{U}(N_3)$ which has a natural realization as the world-volume theory on a superposition of fractional D-branes in the orbifold $\mathbb{C}^3/(\mathbb{Z}_2\times\mathbb{Z}_2)$. The field content of this model is summarized in the quiver diagram of figure 1 and consists of four vector multiplets V^I ($I = 0,1,2,3$), one for each factor of the gauge group, and twelve chiral multiplets Φ^{IJ} (with $I \neq J$) that transform in the bifundamental representation $(N_I, \overline{N}_J)$ of the $\mathrm{U}(N_I)\times\mathrm{U}(N_J)$ sub-group, together with the corresponding anti-chiral multiplets $\overline{\Phi}^{JI}$ that transform in the $(\overline{N}_I, N_J)$ representation.

[2] Terms like, for example, $(\det C)^4\,(\overline{\varphi}\,F)^4$ or $(\det C)^4\,(\overline{\varphi}\,\overline{\lambda}\overline{\lambda}\,F)^2$ will not be considered since they explicitly break the U(1) R-symmetry of the theory.

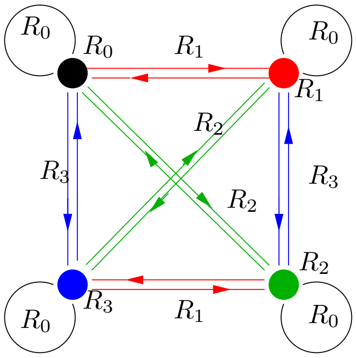

Figure 1: A quiver diagram encodes the field content and the charges of a system of matter-coupled gauge theories. Each dot (labeled by $I = 0,1,2,3$ in this case) corresponds to a $\mathrm{U}(N_I)$ gauge group, for which a gauge multiplet is considered. An oriented link from the I-th to the J-th dot corresponds to a chiral multiplet Φ^{IJ} transforming in the $(N_I, \overline{N}_J)$ representation. As we will discuss in section 3, from a string theory point of view this particular quiver diagram describes a system of fractional D-branes of type-IIB in the orbifold $\mathbb{C}^3/(\mathbb{Z}_2 \times \mathbb{Z}_2)$. In this context the dots represent the fractional branes and the lines the strings stretching between them. For each type of string we indicate the representation of the orbifold group in which the vertex operators should transform in order to survive the orbifold projection.

A NAC deformation of the superspace induces several changes in this quiver theory, which we now analyze. First of all, since the chiral and anti-chiral superfields are in bifundamental representations, the transformation rules (2.16) must be replaced by

$$\begin{aligned} \Phi^{IJ} &\to \Phi'^{IJ} = \mathrm{e}^{-\mathrm{i}\,\Xi_I} \star \Phi^{IJ} \star \mathrm{e}^{\mathrm{i}\,\Xi_J} \\ \overline{\Phi}^{JI} &\to \overline{\Phi}'^{JI} = ee^{-\mathrm{i}\,\overline{\Xi}_J} \star \overline{\Phi}^{JI} \star \mathrm{e}^{\mathrm{i}\,\overline{\Xi}_I} \end{aligned} \tag{2.25}$$

where Ξ_I and $\overline{\Xi}_J$ are defined as in (2.3). Then, if we require that these formulas account for the appropriate gauge transformations of the components, it is necessary to parameterize the superfields as follows

$$\Phi^{IJ} = \varphi^{IJ}(y) + \sqrt{2}\,\theta\chi^{IJ}(y) + \theta\theta\, F^{IJ}(y) \tag{2.26}$$

$$\overline{\Phi}^{JI} = \overline{\varphi}^{JI}(y) - 2\mathrm{i}\,\theta\sigma^\mu\bar\theta\,\partial_\mu\overline{\varphi}^{JI}(y) - \theta\theta\,\bar\theta\bar\theta\,\partial^2\overline{\varphi}^{JI}(y) + \tag{2.27}$$

$$+\sqrt{2}\,\bar\theta\overline{\chi}^{JI}(y) - 2\sqrt{2}\mathrm{i}\,\theta\sigma^\mu\bar\theta\,\partial_\mu\overline{\chi}^{JI}(y) + \bar\theta\bar\theta\,\widetilde{F}^{JI}(y) \tag{2.28}$$

where $\widetilde{F}^{JI}$ is given by the obvious generalization of (2.17), i.e.

$$\begin{aligned} \widetilde{F}^{JI} = {} & \overline{F}^{JI} + 2\mathrm{i}\,C^{\mu\nu}\,\partial_\mu\big(\overline{\varphi}^{JI}\,A^I_\nu + A^J_\nu\,\overline{\varphi}^{JI}\big) - \mathrm{i}\,C^{\mu\nu}\,\big(\overline{\varphi}^{JI}\,A^I_\mu A^I_\nu + A^J_\mu A^J_\nu\,\overline{\varphi}^{JI}\big) + \\ & +\mathrm{i}\,a\,C^{\mu\nu}\,\big(\overline{\varphi}^{JI}\,F^I_{\mu\nu} + F^J_{\mu\nu}\,\overline{\varphi}^{JI}\big) + b\,\det C\,\big(\overline{\varphi}^{JI}\,\overline{\lambda}^I\,\overline{\lambda}^I + \overline{\lambda}^J\,\overline{\lambda}^J\overline{\varphi}^{JI}\big) - \\ & -2\,C^{\mu\nu}\,A^J_\mu\,\overline{\varphi}^{JI}\,A^I_\nu \end{aligned} \tag{2.29}$$

with $\overline{F}^{JI}$ being the auxiliary field conjugate to F^{IJ}.

The gauge invariant kinetic lagrangian for a quiver theory in the C-deformed superspace is simply given by

$$\mathcal{L}_K = \frac{\mathrm{i}}{8\pi}\sum_I\left[\int d^2\theta\;\tau\,\mathrm{Tr}\Big(\mathcal{W}^I \star \mathcal{W}^I\Big) - \int d^2\bar\theta\;\bar\tau\,\mathrm{Tr}\Big(\overline{\mathcal{W}}^I \star \overline{\mathcal{W}}^I\Big)\right] +$$

JHEP05(2005)047

$$+\sum_{I\neq J}\int d^2\theta\, d^2\bar\theta\,\mathrm{Tr}\Big(\overline{\Phi}^{JI}\star \mathrm{e}^{V^I}\star\Phi^{IJ}\star \mathrm{e}^{-V^J}\Big) \tag{2.30}$$

where $\mathcal{W}^I$ is the field-strength superfield for the I-th node of the quiver diagram. Working out the $\star$-products and expanding in θ, after a lengthy but straightforward calculation, one can find the component form of $\mathcal{L}_K$, which turns out to be a natural generalization of what we presented in the previous sub-section for the U(N) theory. It is important to realize that in the quiver case we can add to the lagrangian also a gauge invariant superpotential term given by

$$\mathcal{L}_W+\mathcal{L}_{\overline{W}}=\frac{g}{3}\sum_{I\neq J\neq K}\left[\int d^2\theta\,\mathrm{Tr}\Big(\Phi^{IJ}\star\Phi^{JK}\star\Phi^{KI}\Big)+\int d^2\bar\theta\,\mathrm{Tr}\Big(\overline{\Phi}^{IJ}\star\overline{\Phi}^{JK}\star\overline{\Phi}^{KI}\Big)\right] \tag{2.31}$$

where the sum over the triples $I\neq J\neq K$ describes in fact a sum over all possible triangles of the diagram in figure 1 and the factor of 1/3 eliminates the overcounting of cyclically symmetric terms. The $\star$-products are easily evaluated in the holomorphic part $\mathcal{L}_W$, whose component form is

$$\begin{aligned}\mathcal{L}_W &= g\sum_{I\neq J\neq K}\mathrm{Tr}\left(F^{IJ}\varphi^{JK}\varphi^{KI}-\varphi^{IJ}\chi^{JK}\chi^{KI}\right)+\\ &\quad +g\sum_{I\neq J\neq K}\mathrm{Tr}\left(\frac{1}{4}C^{\mu\nu}\,F^{IJ}\chi^{JK}\sigma^{\mu\nu}\chi^{KI}-\frac{1}{3}\det C\,F^{IJ}F^{JK}F^{KI}\right).\end{aligned} \tag{2.32}$$

The anti-holomorphic piece $\mathcal{L}_{\overline{W}}$ is instead much more involved due to the non-trivial parameterization of the anti-chiral superfields $\overline{\Phi}^{IJ}$ given in (2.28) and (2.29). Finding its complete component expression is just a matter of lengthy algebra; however it is not difficult to see that, among the many C-dependent terms, $\mathcal{L}_{\overline{W}}$ contains the following one

$$2g\sum_{I\neq J\neq K}\mathrm{Tr}\Big(C^{\mu\nu}\,\overline{\varphi}^{IJ}D_\mu\overline{\varphi}^{JK}D_\nu\overline{\varphi}^{KI}\Big) \tag{2.33}$$

whose origin can be simply traced in the $C^{\alpha\beta}$-term of the $\star$-product definition (see eq. (2.10)).

In the following we will show that all structures of the NAC quiver gauge theory we have presented here are reproduced in a natural and efficient way by the dynamics of fractional D-branes in a graviphoton R-R background.

3. $\mathcal{N}=1$ gauge theories from open strings in $\mathbb{C}^3/(\mathbb{Z}_2\times\mathbb{Z}_2)$

It is well-known that quiver gauge theories [26] such as the one considered in section 2 may be derived from a consistent string theory construction; in fact they describe the dynamics of massless modes of the open strings attached to systems of fractional branes in a space whose "internal" directions are orbifolded by some discrete group. The type of orbifold one takes determines the amount of residual supersymmetry and the shape of the quiver. Indeed, in the stringy interpretation, the nodes of the quiver correspond to the various types of fractional branes, which in turn correspond to the irreducible representations of the orbifold group [27]. To engineer a gauge theory in four dimensions we will consider stacks of parallel D3 branes, and mod out the six-dimensional transverse space by the action of a discrete SU(3) subgroup in order to remain with four real supercharges, i.e. with $\mathcal{N} = 1$ supersymmetry; in particular we will consider the $\mathbb{C}^3/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orbifold which yields exactly the quiver of figure 1.

3.1 The $\mathbb{C}^3/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orbifold and itsconformal fields

Let us consider type-IIB string theory in $\mathbb{R}^4 \times \mathbb{C}^3/(\mathbb{Z}_2 \times \mathbb{Z}_2)$. To define the orbifold, we first complexify the "internal" coordinates $x^a \equiv x^5, \ldots, x^{10}$ and the corresponding string fields X^a and ψ^a by setting

$$\begin{aligned} Z^1 &= \frac{(X^5 + \mathrm{i}X^6)}{\sqrt{2}}, & \Psi^1 &= \frac{(\psi^5 + \mathrm{i}\psi^6)}{\sqrt{2}}, \\ Z^2 &= \frac{(X^7 + \mathrm{i}X^8)}{\sqrt{2}}, & \Psi^2 &= \frac{(\psi^7 + \mathrm{i}\psi^8)}{\sqrt{2}}, \\ Z^3 &= \frac{(X^9 + \mathrm{i}X^{10})}{\sqrt{2}}, & \Psi^3 &= \frac{(\psi^9 + \mathrm{i}\psi^{10})}{\sqrt{2}}. \end{aligned} \tag{3.1}$$

Then, we mod out the action of a $\mathbb{Z}_2 \times \mathbb{Z}_2 \subset \mathrm{SO}(6)$ group generated by

$$g_1 = \mathrm{e}^{\mathrm{i}\pi(J_2 - J_3)}, \qquad g_2 = \mathrm{e}^{\mathrm{i}\pi(J_1 - J_3)} \tag{3.2}$$

where $J_{1,2,3}$ are the generators of rotations in the 5-6, 7-8 and 9-10 planes respectively. Explicitly, we have

$$\begin{aligned} g_1 &: \ (Z^1, Z^2, Z^3) \to (Z^1, -Z^2, -Z^3), \\ g_2 &: \ (Z^1, Z^2, Z^3) \to (-Z^1, Z^2, -Z^3), \end{aligned} \tag{3.3}$$

and similarly for $\Psi^{1,2,3}$.

We may summarize the transformation properties (3.3) for the conformal fields ∂Z^i and Ψ^i $(i = 1, 2, 3)$ in the Neveu-Schwarz sector by means of the following table:

conf. field	irrep
∂Z^i, Ψ^i	R_i

(3.4)

where $\{R_I\} = \{R_0, R_i\}$ are the irreducible representations of $\mathbb{Z}_2 \times \mathbb{Z}_2$, identified by writing the character table of the group

	e	g_1	g_2	$g_1 g_2$
R_0	1	1	1	1
R_1	1	1	−1	−1
R_2	1	−1	1	−1
R_3	1	−1	−1	1

(3.5)

The Clebsh-Gordan series for these representations is simply given by

$$R_0 \otimes R_I = R_I\,, \qquad R_i \otimes R_j = \delta_{ij} R_0 + |\epsilon_{ijk}| R_k\,, \tag{3.6}$$

and will be crucial in determining the open string spectrum.

To analyze the Ramond sector, we must consider the action of the orbifold group on spin fields and spinor states. The spin fields are best described within the bosonized version of the so(6) current algebra generated by the world-sheet fermions. So we set

$$\Psi^i = c_i\ \mathrm{e}^{\pm \mathrm{i}\,\varphi_i}\,, \qquad \overline{\Psi}^i = c_i\ \mathrm{e}^{-\mathrm{i}\,\varphi_i} \tag{3.7}$$

where c_i are cocycle factors needed to maintain the fermionic statistic, and φ_i are free bosons with propagators $\langle \varphi_i(z)\varphi_j(z)\rangle = -\delta_{ij}\log(z-w)$. The currents corresponding to the Cartan generators are

$$J_i = -\mathrm{i} :\psi^{3+2i}\psi^{4+2i} :=: \Psi^i\overline{\Psi}^i := \mathrm{i}\,\partial\varphi_i\,, \tag{3.8}$$

while the spin fields $S^A \sim \mathrm{e}^{\mathrm{i}\vec{\lambda}^A\cdot\vec{\varphi}}$ are associated to the so(6) spinor weights

$$\vec{\lambda}^A = \frac{1}{2}(\pm,\pm,\pm)\,, \qquad (A = 1,\ldots,8)\,. \tag{3.9}$$

Using this information, we easily deduce from (3.2) the transformation properties of the various spin fields under the orbifold generators which are summarized in the following table

anti-chiral	chiral	g_1	g_2	irrep
$S^{---} \equiv \mathrm{e}^{-\frac{\mathrm{i}}{2}(\varphi_1+\varphi_2+\varphi_3)}$	$S^{+++} \equiv \mathrm{e}^{\frac{\mathrm{i}}{2}(\varphi_1+\varphi_2+\varphi_3)}$	1	1	R_0
$S^{-++} \equiv \mathrm{e}^{\frac{\mathrm{i}}{2}(-\varphi_1+\varphi_2+\varphi_3)}$	$S^{+--} \equiv \mathrm{e}^{\frac{\mathrm{i}}{2}(\varphi_1-\varphi_2-\varphi_3)}$	1	-1	R_1
$S^{+-+} \equiv \mathrm{e}^{\frac{\mathrm{i}}{2}(\varphi_1-\varphi_2+\varphi_3)}$	$S^{-+-} \equiv \mathrm{e}^{\frac{\mathrm{i}}{2}(-\varphi_1+\varphi_2-\varphi_3)}$	-1	1	R_2
$S^{++-} \equiv \mathrm{e}^{\frac{\mathrm{i}}{2}(\varphi_1+\varphi_2-\varphi_3)}$	$S^{--+} \equiv \mathrm{e}^{\frac{\mathrm{i}}{2}(-\varphi_1-\varphi_2+\varphi_3)}$	-1	-1	R_3

(3.10)

In the sequel, we will need also the transformation properties of the conformal operators corresponding to the roots of so(6), which will play the role of *auxiliary* fields for various $\mathcal{N}=1$ multiplets in the field theory. Recalling that the twelve root vectors of so(6) are $(0,\pm,\pm)$, $(\pm,0,\pm)$, $(\pm,\pm,0)$, from (3.2) we find

current	g_1	g_2	irrep
$\mathrm{e}^{\mathrm{i}(\pm\varphi_2\pm\varphi_3)}$	1	-1	R_1
$\mathrm{e}^{\mathrm{i}(\pm\varphi_1\pm\varphi_3)}$	-1	1	R_2
$\mathrm{e}^{\mathrm{i}(\pm\varphi_1\pm\varphi_2)}$	-1	-1	R_3

(3.11)

Notice that these twelve currents correspond to operators of the form $\Psi^i\Psi^j$, $\Psi^i\overline{\Psi}^j$ and their complex conjugate with $i \neq j$.

The transformation properties under the orbifold group of the various conformal fields determine which states of the string spectrum survive the projection. For open strings, however, one has to take into account also the behaviour of the boundary conditions under the orbifold group. The irreducible consistent boundary conditions for open strings are known as fractional branes and are classified by the irreducible representations of the orbifold group. This means that the endpoint of an open string attached to a fractional brane of type I transforms in the representation R_I. Therefore, to determine which states in the spectrum of an open string stretching between branes of type I and J are invariant, it is necessary to look for trivial factors in the decomposition of $R_I \otimes R_J \otimes Q$, where Q is the representation acting on the string fields, as indicated in the tables (3.4), (3.10) and (3.11). Hence, given the Chan-Paton representations R_I and R_J for the endpoint, the conformal fields creating an invariant state must transform only in certain representations, and all this information is efficiently encoded in a quiver diagram, like the one in figure 1.

3.2 The gauge multiplets

Let us consider a string attached with both ends to branes of the same type, say I. This means that its endpoints do not transform under the orbifold group, since $R_I \otimes R_I = R_0$ as one can see from (3.6). Therefore also the oscillator part of any surviving state must be invariant under the orbifold. For example, in the NS sector the states $\psi^{\mu}_{-\frac{1}{2}}|0\rangle$ survive, but none of the states $\Psi^{i}_{-\frac{1}{2}}|0\rangle$ does.

More generally, given a stack of N_I branes of type I, the massless open string excitations organize in a $\mathcal{N}=1$ vector multiplet for the group $\mathrm{U}(N_I)$ produced by the following vertex operators

$$V_A(p) = A_\mu(p)\,\frac{\psi^\mu}{\sqrt{2}}\,\mathrm{e}^{-\phi}\,\mathrm{e}^{\mathrm{i}p\cdot X} \tag{3.12}$$

in the NS sector, and[3]

$$\begin{aligned} V_\lambda(p) &= \mathrm{i}\,\lambda^\alpha(p)\,S_\alpha\,S^{---}\,\mathrm{e}^{-\frac{1}{2}\phi}\,\mathrm{e}^{\mathrm{i}p\cdot X}\,, \\ V_{\overline{\lambda}}(p) &= \overline{\lambda}_{\dot\alpha}(p)\,S^{\dot\alpha}\,S^{+++}\,\mathrm{e}^{-\frac{1}{2}\phi}\,\mathrm{e}^{\mathrm{i}p\cdot X} \end{aligned} \tag{3.13}$$

in the R sector, with S_α and $S^{\dot\alpha}$ being the chiral and anti-chiral spin fields along the world-volume directions. The polarizations A_μ, λ^α and $\overline{\lambda}_{\dot\alpha}$ carry Chan-Paton indices in the adjoint representation of $\mathrm{U}(N_I)$. In the following, we will adopt the same notation of section 2, and use A^I_μ, λ^I_α and $\overline{\lambda}^I_{\dot\alpha}$ to denote the gauge multiplet living on fractional branes of type I. In writing the vertex operators (3.12) and (3.13) we have set $2\pi\alpha' = 1$, and we will consistently do so henceforth. Appropriate powers of $2\pi\alpha'$ can be easily reinstated in our formulas so as to give A_μ dimensions of $(\text{length})^{-1}$, and to the gauginos λ_α and $\overline{\lambda}_{\dot\alpha}$ dimensions of $(\text{length})^{-3/2}$.

[3]Comparing with ref. [25], we have included a factor of i in the vertex of the gluino λ in order to be consistent with the notation of section 2.

As a matter of fact, also the auxiliary field D of the $\mathcal{N}=1$ vector multiplet admits a stringy realization. In fact, it can be associated to the following (non-BRST invariant) vertex in the 0-superghost picture of the NS sector [28]

$$V_D(p) = \frac{1}{3}\, D(p)\, \delta_{ij} : \Psi^i\, \overline{\Psi}^j : \mathrm{e}^{ip\cdot X} = \frac{2\mathrm{i}}{3}\, D(p) \Big(\sum_i \partial\varphi_i \Big)\, \mathrm{e}^{\mathrm{i}p\cdot X} \,. \tag{3.14}$$

The vertices we introduced above are connected with each other through the action of the supersymmetry charges

$$Q_\alpha = \oint \frac{dz}{2\pi\mathrm{i}}\, j_\alpha(z) \qquad \text{and} \qquad \overline{Q}_{\dot\alpha} = \oint \frac{dz}{2\pi\mathrm{i}}\, j_{\dot\alpha}(z) \tag{3.15}$$

where the currents (in the $(-\frac{1}{2})$-picture) are given by

$$j_\alpha(z) = S_\alpha(z)\, S^{---}(z)\, \mathrm{e}^{-\frac{1}{2}\phi(z)} \,, \qquad \overline{j}_{\dot\alpha}(z) = S_{\dot\alpha}(z)\, S^{+++}(z)\, \mathrm{e}^{-\frac{1}{2}\phi(z)} \,. \tag{3.16}$$

For instance, it is easy to see that

$$\begin{aligned} [\xi Q \,,\, V_D(w;p)] &= \xi^\alpha \oint_w \frac{dz}{2\pi\mathrm{i}}\, j_\alpha(z)\, V_D(w;p) \\ &= -\, \xi^\alpha\, D(p)\, S_\alpha(w)\, S^{---}(w)\, \mathrm{e}^{-\frac{1}{2}\phi(w)}\, \mathrm{e}^{ip\cdot X(w)} \,. \end{aligned} \tag{3.17}$$

Upon comparison with (3.13), we recognize in the the last line the vertex operator of a gaugino, and thus we can rewrite (3.17) as

$$[\xi Q \,,\, V_D(w;p)] = V_{\delta\lambda}(w;p) \tag{3.18}$$

with $\delta\lambda = \mathrm{i}\,\xi D$, in agreement with the standard definitions in supersymmetric field theory (see eq. (2.22)). With similar calculations one can reconstruct also the other terms in the supersymmetry transformations of the $\mathcal{N}=1$ gauge multiplet.

3.3 The chiral multiplets

The massless spectrum of open strings stretching between a fractional brane of type I and one of type J is produced by vertex operators which transform in some non-trivial representation of the orbifold group, as indicated by the quiver diagram in figure 1. Let us consider, for example, the oriented open strings stretching between branes of type 0 and type 1. Then, from (3.6) we see that the vertices surviving the orbifold projection must transform in the representation R_1. At the massless level we find

$$\begin{aligned} V_{\varphi^{01}}(p) &= \frac{g}{2}\, \varphi^{01}(p)\, \overline{\Psi}^1\, \mathrm{e}^{-\phi}\, \mathrm{e}^{\mathrm{i}p\cdot X} \,, \\ V_{F^{01}}(p) &= g\, F^{01}(p)\, \Psi^2 \Psi^3\, \mathrm{e}^{\mathrm{i}p\cdot X} \end{aligned} \tag{3.19}$$

in the NS sector, and

$$V_{\chi^{01}}(p) = \frac{g}{\sqrt{2}}\, \chi^{01\,\alpha}(p)\, S_\alpha\, S^{-++}\, \mathrm{e}^{-\frac{1}{2}\phi}\, \mathrm{e}^{ip\cdot X} \tag{3.20}$$

in the R sector. These are precisely the vertices for the fields (including the auxiliary one) of a chiral supermultiplet which, following the notation of section 2, we organize in the superfield Φ^{01}. The polarizations in (3.19) and (3.20) carry a superscript ^{01} which specifies the boundary conditions of the open strings under consideration, and the normalizations of the vertex operators are fixed in order to obtain the canonical action in the field theory limit, as we shall see later.

Target-space supersymmetry connects the above vertices among each other in the standard way. Indeed, in analogy with (3.17), one can show, for example, that

$$[\xi Q\,,\, V_{F^{01}}(p)] = V_{\delta\chi^{01}}(p)\,, \tag{3.21}$$

where $\delta\chi^{01} = \sqrt{2}\,\xi F^{01}$ which exactly agrees with (2.22).

By construction, the chiral superfield Φ^{01} transforms in the bifundamental representation $(N_0, \overline{N}_1)$ with respect to the $\mathrm{U}(N_0)\times\mathrm{U}(N_1)$ gauge groups defined on the fractional branes of type 0 and 1 respectively.[4] Its complex conjugate is denoted as $\overline{\Phi}^{10}$ and corresponds to open strings oriented from branes of type 1 to branes of type 0 and transforming in the $(\overline{N}_0, N_1)$ representation. Notice that there exists also another *independent* chiral multiplet arising from the strings oriented from branes of type 1 to branes of type 0, and transforming in the $(\overline{N}_0, N_1)$ representation. This other multiplet is denoted as Φ^{10} and the vertex operators for its component fields have the same form as those in (3.19) and (3.20), since they must obey again the requirement of belonging to the representation R_1 of the orbifold group. Altogether, from a generic system of fractional branes in the orbifold $\mathbb{C}^3/(\mathbb{Z}_2\times\mathbb{Z}_2)$, we find twelve chiral multiplets Φ^{IJ} (with $I \neq J = 0,\ldots,3$) and their complex conjugates, that are associated to the various oriented links of the quiver diagram in figure 1.

3.4 Effective lagrangians

The effective lagrangians for the massless multiplets of the quiver theory can be derived by taking the field theory limit $\alpha' \to 0$ of string scattering amplitudes involving the vertex operators introduced before. For example, for the gauge multiplet of type I, we must consider diagrams with the vertex operators (3.12), (3.13) and (3.14) emitted from disks whose boundaries are entirely attached to branes of type I. In the field theory limit these amplitudes lead to[5]

$$\mathcal{L}_{\rm gauge} = \frac{1}{g^2}\,\mathrm{Tr}\bigg\{\frac{1}{2}\,(F^I_{\mu\nu})^2 - 2\mathrm{i}\,\bar\lambda^I\bar\sigma^\mu D_\mu\lambda^I - (D^I)^2\bigg\}\,. \tag{3.22}$$

By introducing a self-dual antisymmetric auxiliary field $H^I_{\mu\nu}$, it is possible to reduce the quartic interactions in $\mathrm{Tr}(F^I_{\mu\nu})^2$ to cubic ones. Indeed, the lagrangian

$$\mathcal{L}'_{\rm gauge} = \frac{1}{g^2}\,\mathrm{Tr}\bigg\{\big(\partial_\mu A^I_\nu - \partial_\nu A^I_\mu\big)\partial_\mu A^I_\nu + 2\mathrm{i}\,\partial_\mu A^I_\nu\big[A^I_\mu, A^I_\nu\big] - (D^I)^2 -$$

[4]Explicitly, Φ^{01} is a $N_0\times N_1$ complex matrix $\left(\Phi^{01}\right)^{i_0}_{\;i_1}$ where $i_0 = 1,\ldots,N_0$ and $i_1 = 1,\ldots,N_1$. In the sequel we will adopt a matrix notation without explicit use of indices, so we'll have to care about the ordering. For example the covariant derivatives are $D_\mu\varphi^{01} = \partial_\mu\varphi^{01} + \mathrm{i}A^0_\mu\varphi^{01} - \mathrm{i}\varphi^{01}A^1_\mu$.

[5]Various details on these calculations can be found, e.g., in ref. [25].

$$- 2\mathrm{i}\,\bar{\lambda}^I \bar{\sigma}^\mu D_\mu \lambda^I + \frac{1}{4}\,(H^I_{\mu\nu})^2 + H^I_{\mu\nu}\left[A^I_\mu, A^I_\nu\right]\Big\} \tag{3.23}$$

is easily seen to be equivalent to (3.22) after integrating out $H^I_{\mu\nu}$ through its algebraic equations of motion. The auxiliary field $H^I_{\mu\nu}$ admits a stringy representation in terms of the following (non-BRST invariant) vertex operator in the 0-superghost picture [25]

$$V_{H_{\mu\nu}}(p) = \frac{1}{2}\,H^I_{\mu\nu}(p)\;:\psi^\nu\psi^\mu:\;\mathrm{e}^{\mathrm{i}p\cdot X}\,. \tag{3.24}$$

Notice that the structure of this vertex is the same as that of the $p\cdot\psi\psi^\mu$ part of the (properly normalized) vertex for the gauge field in the 0-picture, namely

$$V_A(p) = 2\mathrm{i}\,A^I_\mu(p)\,(\partial X^\mu + \mathrm{i}\,p\cdot\psi\,\psi^\mu)\,\mathrm{e}^{\mathrm{i}p\cdot X}\,. \tag{3.25}$$

Thus, whenever in a disk amplitude we get a non-vanishing amplitude by inserting a vertex in the 0-picture for A^I_μ, we get also a non-vanishing amplitude by inserting the vertex (3.24) for $H^I_{\mu\nu}$.

It is worth pointing out that this auxiliary field is useful not only to reduce the quartic interactions in the gauge lagrangian to cubic ones, but also to linearize the supersymmetry transformations of the vector multiplet. For example, by using the vertices (3.25) and (3.24) and computing

$$\left[\xi Q\,,\,V_A(p)\right] \qquad \text{and} \qquad \left[\xi Q\,,\,V_{H_{\mu\nu}}(p)\right] \tag{3.26}$$

one easily obtains the following supersymmetry transformation for the gaugino

$$\delta\lambda = -\xi\,\sigma^{\mu\nu}\left(\partial_\mu A_\nu - \frac{\mathrm{i}}{4}\,H_{\mu\nu}\right) \tag{3.27}$$

which, upon eliminating $H_{\mu\nu}$ through its field equation, becomes $\delta\lambda = -\frac{1}{2}\,\xi\,\sigma^{\mu\nu}F_{\mu\nu}$, with the non-linear terms of the field strength included (see eq. (2.22)).

A similar analysis can be done also in the matter sector. By computing all disk diagrams with insertions of the vertex operators (3.19) and (3.20), and then taking the field theory limit, one may reconstruct the effective action for the chiral multiplet Φ^{01} (and its complex conjugate $\overline{\Phi}^{10}$) which is given by

$$\begin{aligned}\mathcal{L}_{\mathrm{matt}} = \mathrm{Tr}\Big\{ & D_\mu\overline{\varphi}^{10} D_\mu \varphi^{01} - \mathrm{i}\,\overline{\chi}^{10}\bar{\sigma}^\mu D_\mu \chi^{01} + \overline{F}^{10} F^{01} + \\ & + \overline{\varphi}^{10} D^0 \varphi^{01} - \varphi^{01} D^1 \overline{\varphi}^{10} + \sqrt{2}\,\mathrm{i}\,(\overline{\chi}^{10}\overline{\lambda}^0\varphi^{01} - \varphi^{01}\overline{\lambda}^1\overline{\chi}^{10}) + \\ & + \sqrt{2}\,\mathrm{i}\,(\overline{\varphi}^{10}\lambda^0\chi^{01} - \chi^{01}\lambda^1\overline{\varphi}^{10})\Big\}\,.\end{aligned} \tag{3.28}$$

Notice that the disk diagrams which lead to this effective lagrangian have their boundaries lying partly on branes of type 0 and partly on branes of type 1, and consequently the gauge fields can be of either type.

As for the gauge sector, also in the matter part it is possible to introduce suitable auxiliary fields to decouple the quartic interactions coming from the covariant derivatives of the scalars. Indeed, the very first term of eq. (3.28) can be rewritten as

$$\mathrm{Tr}\left\{\partial_\mu\overline{\varphi}^{10}\partial_\mu\varphi^{01} + \overline{H}^{10}_\mu H^{01}_\mu + \mathrm{i}\big(\partial_\mu\overline{\varphi}^{10} - \mathrm{i}\,\overline{H}^{10}_\mu\big)\big(A^0_\mu\varphi^{01} - \varphi^{01}A^1_\mu\big) + \right.$$

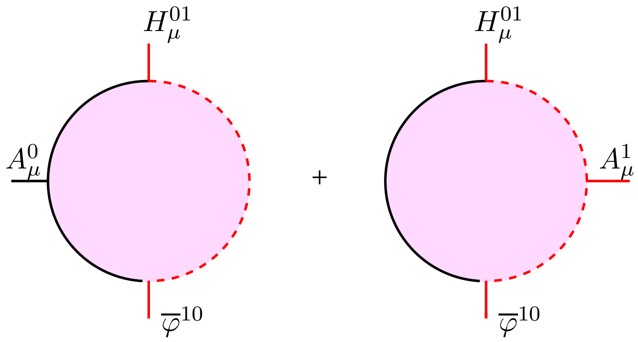

Figure 2: The diagrams accounting for the interactions of the auxiliary field H^{01}_μ.

$$+\,\mathrm{i}\big(A^1_\mu\overline{\varphi}^{10} - \overline{\varphi}^{10}A^0_\mu\big)\big(\partial_\mu\varphi^{01} - \mathrm{i}\,H^{01}_\mu\big)\Big\}\,, \tag{3.29}$$

i.e. with only cubic interactions. The new H-dependent terms arise from disk diagrams with insertions of the following auxiliary vertex operator

$$V_{H^{01}_\mu}(p) = g\,H^{01}_\mu(p)\,\psi^\mu\,\overline{\Psi}^1\,\mathrm{e}^{\mathrm{i}p\cdot X} \tag{3.30}$$

and of the corresponding complex conjugate, as shown in figure 2.

Notice that this vertex is identical to the fermionic part of the (properly normalized) scalar vertex in the 0-superghost picture, namely

$$V_{\varphi^{01}}(p) = \sqrt{2}\mathrm{i}\,g\,\varphi^{01}(p)\left(\partial\overline{Z}^1 + \mathrm{i}p\cdot\psi\,\overline{\Psi}^1\right)\mathrm{e}^{\mathrm{i}p\cdot X}\,. \tag{3.31}$$

Thus everywhere we have to consider a diagram with a vertex for φ^{01} in the 0-picture (which produces terms containing $\partial_\mu\varphi^{01}$ in the lagrangian), we have also to consider a diagram with the auxiliary vertex (3.30). The net outcome of this is that all occurrences of $\partial_\mu\varphi^{01}$ in the effective action are promoted to $\big(\partial_\mu\varphi^{01} - \mathrm{i}H^{01}_\mu\big)$, which in turn becomes the complete covariant derivative $D_\mu\varphi^{01}$ after integrating out H^{01}_μ through its field equation.

Let us now turn to the superpotential. Whenever the string configuration contains at least three different types of branes, then the Chan-Paton structure of the vertex operators for chiral multiplets allows for a cubic holomorphic superpotential. Let us suppose, for example, to have branes of type 0, 3 and 1. Then, if consider a disk diagram with three vertices corresponding to some of the fields in the multiplets Φ^{03}, Φ^{31} and Φ^{10}, taken in this order, we have the possibility of getting a non-vanishing result. Indeed, the disk boundary jumps first from type 0 to type 3, then from 3 to 1 and finally returns back to type 0 to close in a consistent way. Of course, we could get a non-zero amplitude also by inserting the vertices corresponding to Φ^{01}, Φ^{13} and Φ^{30}, i.e. by following the triangle on the quiver diagram in the opposite direction, or by utilizing the anti-chiral counterparts of the above possibilities which lead to a cubic anti-holomorphic superpotential.[6]

[6]Notice that the Chan-Paton structure allows in principle configurations that involve both holomorphic and anti-holomorphic fields, like for example Φ^{03}, $\overline{\Phi}^{31}$ and Φ^{10}. However, the corresponding amplitudes vanish since the vertex operators in these configurations do not saturate the charges with respect to the internal world-sheet bosons φ_i. Thus, only holomorphic or anti-holomorphic superpotentials are possible.

JHEP05(2005)047

Specifically, if we compute the amplitude among $V_{F^{03}}$, $V_{\varphi^{31}}$ and $V_{\varphi^{10}}$ and take the field theory limit, we obtain the following term in the effective lagrangian

$$g\ \mathrm{Tr}\left(F^{03}\varphi^{31}\varphi^{10}\right), \tag{3.32}$$

which is related by supersymmetry to the Yukawa term

$$-g\,\mathrm{Tr}\left(\varphi^{03}\chi^{31}\chi^{10}\right) \tag{3.33}$$

arising from the amplitude among $V_{\varphi^{03}}$, $V_{\chi^{31}}$ and $V_{\chi^{10}}$ (see also the first line of eq. (2.32)). These interactions as well all others corresponding to different combinations of fields can be summarized in a holomorphic superpotential of the form

$$W = \frac{g}{3}\sum_{I\neq J\neq K}\mathrm{Tr}\left(\Phi^{IJ}\Phi^{JK}\Phi^{KI}\right) \tag{3.34}$$

or in its anti-holomorphic counterpart.

4. NAC deformation from R-R flux

We now analyze the deformations of the $\mathcal{N}=1$ quiver theory discussed in the previous section that are induced by a non-trivial R-R flux corresponding to a graviphoton background with constant field strength. This background is described by a constant antisymmetric tensor $C_{\mu\nu}$ which we take to be self-dual and which is responsible of the NAC deformation of the $\mathcal{N}=1$ superspace. From the string point of view, $C_{\mu\nu}$ is a R-R field strength, and more precisely it is the R-R 5-form of type-IIB string theory,[7] wrapped around the internal orbifold space and described by the following closed string vertex operator (in the $(-1/2,-1/2)$ superghost picture)

$$V_C(z,\overline{z}) = \frac{1}{4\pi^2}\,C^{\alpha\beta}\,S_\alpha(z)\,S^{---}(z)\,\mathrm{e}^{-\frac{1}{2}\phi(z)}\,\widetilde{S}_\beta(\overline{z})\,\widetilde{S}^{---}(\overline{z})\,\mathrm{e}^{-\frac{1}{2}\tilde{\phi}(\overline{z})}\,. \tag{4.1}$$

Here, using the arguments explained in ref. [25], we have already identified the symmetric bispinor polarization of V_C with the non-anti-commutativity parameter $C^{\alpha\beta}$ used in section 2. It is worth recalling that such parameter has dimensions of (length), and thus a factor of $(2\pi\alpha')^{-1/2}$ should be included in right hand side of (4.1) to make V_C adimensional. Even if we are using conventions in which $2\pi\alpha'=1$, these dimensional considerations will be crucial in the following. In (4.1) the tilde denotes the right movers of the closed string, and z a point in the upper-half complex plane, which is conformally equivalent to the interior of a disk. Notice that the vertex operator V_C does not contain the usual plane wave term $\mathrm{e}^{ip\cdot X}$, since we are considering a *constant* background with $p=0$.

We are now going to systematically study string amplitudes for fractional D3 branes of the orbifold $\mathbb{C}^3/(\mathbb{Z}_2\times\mathbb{Z}_2)$ in the presence of the non-trivial R-R background (4.1), i.e. we shall compute mixed open/closed string amplitudes on disks with different types boundary

[7] The effect of a constant RR 5-form field-strength background in the $\mathcal{N}=4$ case has been recently considered in [29].

conditions corresponding to the various types of branes. In any mixed open/closed string amplitudes on a disk, the presence of the boundary forces an identification between left- and right-moving oscillators of the closed strings. In our specific case, this identification is the same on all types of fractional D3 branes, and amounts, in practice, to the following replacements (see e.g. ref. [25] for details)

$$\widetilde{S}_\alpha(\overline{z}) \to S_\alpha(\overline{z})\,, \qquad \widetilde{S}^{---}(\overline{z}) \to \widetilde{S}^{---}(\overline{z})\,, \qquad \widetilde{\phi}(\overline{z}) \to \phi(\overline{z})\,. \tag{4.2}$$

As a consequence, we see that any insertion of the R-R vertex (4.1) carries an effective charge $(-1,-1,-1)$ with respect to the three world-sheet scalars φ_1, φ_2 and φ_3 which bosonize the fermions in the internal orbifold directions, and an effective superghost charge (-1). Therefore, in order to have a non vanishing amplitude with a single R-R insertion, we have to choose the open string vertex operators in such a way that they carry an effective total charge $(+1,+1,+1)$ with respect to the internal world-sheet bosons, and an effective total superghost charge (-1). These new requirements add to the one of having a consistent Chan-Paton structure, which we already encountered. Furthermore, we are interested only in amplitudes which survive in the field theory limit $\alpha' \to 0$ with g fixed. Since the factors of $(2\pi\alpha')^h$ (which we have not written explicitly) in the definitions of the vertex operators give space-time dimensions of (length)$^{-2h}$ to the corresponding polarizations, this implies that we have to look only for structures with total dimension of (length)$^{-4}$, including the contribution of the R-R field which carries dimension of (length).

Let us now analyze the effects produced by the insertion of the R-R vertex (4.1) in the various sectors.

4.1 The gauge and chiral matter sectors

The study of the R-R deformation in the pure gauge sector has been the subject of ref. [25], where it was shown that the only string amplitudes on disks with a single type of boundary that do not vanish in the field theory limit, are

$$\langle\!\langle\, V_{\overline{\lambda}^I}\, V_{\overline{\lambda}^I}\, V_{A^I_\mu}\, V_C \,\rangle\!\rangle \qquad \text{and} \qquad \langle\!\langle\, V_{\overline{\lambda}^I}\, V_{\overline{\lambda}^I}\, V_{H^I_{\mu\nu}}\, V_C \,\rangle\!\rangle, \tag{4.3}$$

and correspond to the following contribution to the effective lagrangian for the $\mathrm{U}(N_I)$ gauge fields[8]

$$\frac{4\mathrm{i}}{g^2}\, C^{\mu\nu}\, \mathrm{Tr}\left\{ \left(\partial_\mu A^I_\nu - \frac{\mathrm{i}}{4}\, H^I_{\mu\nu} \right) \overline{\lambda}^I \overline{\lambda}^I \right\}. \tag{4.4}$$

When this term is added to the undeformed lagrangian (3.23) and the auxiliary field $H^I_{\mu\nu}$ is integrated out, one recovers precisely the $\mathcal{N} = 1/2$ gauge lagrangian (2.19) that follows from the NAC deformation of the superspace.

In the case of disks with more than one type of boundary, we have more possibilities. For example a consistent Chan-Paton structure and a correct balance of internal charges can be obtained by inserting V_C together with a vertex for $\overline{\varphi}^{10}$ (with charges $(1,0,0)$),

[8] In ref. [25] we used a $C^{\mu\nu}$ that is twice the one used in the present paper and in the majority of the literature. We have taken this difference into account in writing eq. (4.4).

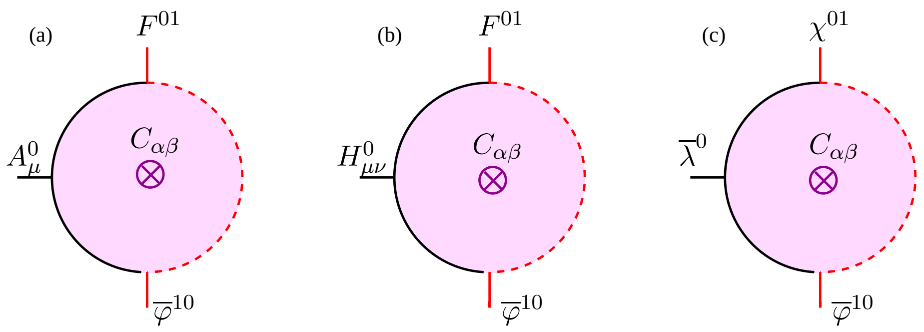

Figure 3: Examples of diagrams with a R-R insertion on a disks with two distinct types of boundaries.

and a vertex for F^{01} (with charges $(0,1,1)$). As far as the superghost background charge is concerned, the vertex for $V_{F^{01}}$ is defined in the 0-picture, and so we have to insert the vertex $V_{\overline{\varphi}^{10}}$ in the (-1)-picture to soak up the superghost number anomaly. However, an amplitude with just $C^{\alpha\beta}$, $\overline{\varphi}^{10}$ and F^{01} cannot survive in the field theory limit, since these fields combined have dimensions of $(\text{length})^{-2}$ and no momentum factor is produced given the above picture assignments. The way out, in this example, is clearly to insert a further vertex in the 0-picture, that carries zero internal charge and supplies the needed mass dimensions. Such vertices may only come from the gauge multiplets of $\mathrm{U}(N_0)$ or $\mathrm{U}(N_1)$, and in principle can be either vertices for the gauge field A_μ, for the D field or for the "auxiliary" fields $H_{\mu\nu}$, which we can insert either on the boundary of type 0 or on the boundary of type 1.

If we insert a vertex for a D field, i.e. if we compute $\langle\!\langle\, V_{\overline{\varphi}^{10}} V_{D^0} V_{F^{01}} V_C \,\rangle\!\rangle$ or $\langle\!\langle\, V_{\overline{\varphi}^{10}} V_{F^{01}} V_{D^1} V_C \,\rangle\!\rangle$, we find that the resulting world-sheet correlator in the SO(4) current algebra sector is

$$\langle S_\alpha(z)\, S_\beta(\overline{z})\rangle \propto \varepsilon_{\alpha\beta} \tag{4.5}$$

which vanishes when it is contracted with the symmetric polarization $C^{\alpha\beta}$. Thus, we have to consider the other two possibilities, which are represented in figure 3*a* and 3*b*.

When we insert the gauge field vertex (3.25), the part containing ∂X^μ does not contribute, as it leads again to the correlator (4.5), while the fermionic part containing $p\cdot\psi\,\psi^\mu$ produces a non-vanishing result. We have

$$\langle\!\langle\, V_{\overline{\varphi}^{10}} V_{A^0} V_{F^{01}} V_C \,\rangle\!\rangle \equiv C_{\text{disk}} \int \frac{\prod_i dy_i\, dz d\overline{z}}{dV_{\text{CKG}}} \langle V_{\overline{\varphi}^{10}}(p_1;y_1)\, V_{A^0}(p_2;y_2)\, V_{F^{01}}(p_3;y_3)\, V_C(z,\overline{z})\rangle \tag{4.6}$$

where $C_{\text{disk}} = 4/g^2$ is the normalization of any disk amplitude in our present conventions (see e.g. ref. [30] for further details) and dV_{CKG} is the $\mathrm{Sl}(2,\mathbb{R})$ invariant volume element. The insertion points y_i of the open string vertices are integrated on the real axis with $y_1 \geq y_2 \geq y_3$, while the closed string insertion z is integrated in the upper half complex plane. More explicitly, the amplitude (4.6) is

$$\frac{\mathrm{i}}{\pi^2}\,\mathrm{Tr}\Big[C^{\alpha\beta}\overline{\varphi}^{10}(p_1)\,\big(\mathrm{i}\,p_2^\mu\, A^0_\nu(p_2)\big)\,F^{01}(p_3)\Big]\int \frac{\prod_i dy_i\, dz d\overline{z}}{dV_{\text{CKG}}}\,\times$$

$$\times\Big\{\langle \mathrm{e}^{-\phi(y_1)}\,\mathrm{e}^{-\frac{1}{2}\phi(z)}\,\mathrm{e}^{-\frac{1}{2}\phi(\overline{z})}\rangle\langle\Psi^1(y_1)\,\Psi^2\Psi^3(y_3)\,S^{---}(z)\,S^{---}(\overline{z})\rangle$$

$$\langle :\psi^\mu\psi^\nu:(y_2)\,S_\alpha(z)\,S_\beta(\overline{z})\rangle\,\langle \mathrm{e}^{\mathrm{i}p_1\cdot X(y_1)}\,\mathrm{e}^{\mathrm{i}p_2\cdot X(y_2)}\,\mathrm{e}^{\mathrm{i}p_3\cdot X(y_3)}\rangle\Big\}\,. \tag{4.7}$$

Using the correlation functions given in appendix A.2, and exploiting the $\mathrm{SL}(2,\mathbb{R})$ invariance to fix $y_1 \to \infty$, $z \to \mathrm{i}$ and $\overline{z} \to -\mathrm{i}$, we are left with an integral over the remaining positions y_2 and y_3, which reads

$$\int_{-\infty}^{+\infty} dy_2 \int_{-\infty}^{y_2} dy_3\,\frac{1}{\left(y_2^2+1\right)\left(y_3^2+1\right)} = \frac{\pi^2}{2}\,. \tag{4.8}$$

Putting everything together, we finally obtain the following contribution to the effective lagrangian

$$2\mathrm{i}\ \mathrm{Tr}\left(C^{\mu\nu}\,\partial_\mu A^0_\nu\,F^{01}\overline{\varphi}^{10}\right). \tag{4.9}$$

Instead of the vertex V_{A^0}, we could have placed a vertex for A^1_μ on the boundary portion of type 1, obtaining, a part from the different ordering of the Chan Paton factors and a different sign, the same result as in eq. (4.9). Moreover, as we already observed, we may obtain a non vanishing amplitude also by replacing the 0-picture vertex for the gauge field with the one for the auxiliary field $H_{\mu\nu}$. This computation of course generalizes to any disk with two types of boundary, and so altogether we get the following contribution to the effective lagrangian

$$2\mathrm{i}\sum_{J\neq I}\mathrm{Tr}\left\{C^{\mu\nu}\left(\partial_\mu A^I_\nu - \frac{\mathrm{i}}{4}\,H^I_{\mu\nu}\right)\left(F^{IJ}\overline{\varphi}^{JI} - \overline{\varphi}^{IJ}F^{JI}\right)\right\}. \tag{4.10}$$

Notice that this term has the same structure as the C-dependent term (4.4) that was already present in the pure gauge sector. Since there are no other diagrams involving the R-R background and the auxiliary field $H^I_{\mu\nu}$, when we add the two contributions (4.4) and (4.10) to the undeformed action (3.23), we find that the auxiliary field can be eliminated through the following equation

$$H^I_{\mu\nu} = -2\left[A^I_\mu\,,\,A^I_\mu\right]^{(+)} - 2\,C_{\mu\nu}\left(\overline{\lambda}^I\overline{\lambda}^I + \frac{g^2}{2}\sum_{J\neq I}\left(F^{IJ}\overline{\varphi}^{JI} - \overline{\varphi}^{IJ}F^{JI}\right)\right) \tag{4.11}$$

where the superscript $^{(+)}$ stands for the self-dual part. Plugging this identification back in the lagrangian, and summing over all types of branes, we find that the deformation terms that must be added the Yang-Mills lagrangian of the quiver theory are

$$\frac{1}{g^2}\sum_I \mathrm{Tr}\Bigg\{2\mathrm{i}\,C_{\mu\nu}\,F^I_{\mu\nu}\left(\overline{\lambda}^I\overline{\lambda}^I + \frac{g^2}{2}\sum_{J\neq I}\left(F^{IJ}\overline{\varphi}^{JI} - \overline{\varphi}^{IJ}F^{JI}\right)\right) -$$

$$- 4\,\det C\left(\overline{\lambda}^I\overline{\lambda}^I + \frac{g^2}{2}\sum_{J\neq I}\left(F^{IJ}\overline{\varphi}^{JI} - \overline{\varphi}^{IJ}F^{JI}\right)\right)^2\Bigg\}. \tag{4.12}$$

So far we have considered disk diagrams with open string vertices in the NS sector. However, there are non-vanishing diagrams involving also fermionic vertices from the R sector. An example of such diagrams is represented in figure 3c which corresponds to the amplitude $\langle\!\langle\, V_{\overline{\varphi}^{10}} V_{\overline{\lambda}^{0}} V_{\chi^{01}} V_C \,\rangle\!\rangle$. To soak up the superghost charge, we put the vertices for $\overline{\lambda}^{0}$ and χ^{01} in the $(-1/2)$-picture and the vertex for $\overline{\varphi}^{10}$ in the 0-picture as given in (3.31). Using the explicit expressions for these vertices and performing the appropriate OPE's, one can easily compute this string amplitude along the same lines discussed above and in the end one finds the following contribution to the effective lagrangian

$$\sqrt{2}\, C^{\mu\nu}\, \mathrm{Tr}\Big(\overline{\lambda}^{0} \bar{\sigma}_{\nu} \chi^{01}\, \partial_{\mu}\overline{\varphi}^{10}\Big)\,. \tag{4.13}$$

Similarly to the case discussed in section 3.3, besides the previous diagram we have also to consider the one where the 0-picture vertex $V_{\overline{\varphi}^{10}}$ is replaced by the vertex for the auxiliary field $\overline{H}_{\mu}^{10}$ given in (3.30), with the result that $\partial_{\mu}\overline{\varphi}^{10}$ in (4.13) is shifted to $\big(\partial_{\mu}\overline{\varphi}^{10} - \mathrm{i}\,\overline{H}_{\mu}^{10}\big)$. Notice that instead there are no amplitudes involving the R-R background and the auxiliary field H_{μ}^{01}, due to unbalanced internal charges. Therefore, when we add these terms to the undeformed lagrangian (3.29), we find that the auxiliary field $\overline{H}_{\mu}^{10}$ can still be eliminated through its undeformed equation of motion, namely

$$\overline{H}_{\mu}^{10} = \overline{\varphi}^{10} A_{\mu}^{0} - A_{\mu}^{1} \overline{\varphi}^{10}\,. \tag{4.14}$$

Again, the net effect is that the ordinary derivative in (4.13) is promoted to the full covariant derivative $D_{\mu}\overline{\varphi}^{10}$ and gauge invariance is restored. Repeating this analysis for all possible multiplets on various types of boundaries, we find that the C-dependent lagrangian arising from fermionic vertices of the R sector is

$$\sqrt{2}\, C^{\mu\nu} \sum_{J\neq I} \mathrm{Tr}\left\{ \big(\overline{\lambda}^{I} \bar{\sigma}_{\nu} \chi^{IJ} - \chi^{IJ} \sigma_{\nu} \overline{\lambda}^{J}\big) D_{\mu}\overline{\varphi}^{JI} \right\}. \tag{4.15}$$

Eqs. (4.12) and (4.15) describe the deformation terms induced by the R-R graviphoton background (4.1) on the effective action of the quiver gauge theory, and are strictly related to those that can be obtained using the NAC $\star$-product deformation described in section 2 (see eq. (2.30)). To make a simple comparison, let us concentrate on a single gauge group, say $\mathrm{U}(N_0)$ which corresponds to the branes of type 0, and on single charged chiral multiplet, say Φ^{01} and its conjugate $\overline{\Phi}^{10}$. Dropping for ease of notation the indices on such fields, we can easily see that (4.12) and (4.15) in this case reduce to

$$\frac{1}{g^2}\mathrm{Tr}\Big\{ 2\mathrm{i}\, C^{\mu\nu} F_{\mu\nu} \Big(\overline{\lambda}\,\overline{\lambda} + \frac{g^2}{2}\, F\,\overline{\varphi}\Big) - 4\, \det C \Big(\overline{\lambda}\,\overline{\lambda} + \frac{g^2}{2}\, F\,\overline{\varphi}\Big)^2 + \sqrt{2}\, g^2\, C^{\mu\nu} D_{\mu}\overline{\varphi}\, \overline{\lambda}\, \bar{\sigma}_{\nu} \chi \Big\}\,. \tag{4.16}$$

These are precisely the interaction terms that appear in the lagrangians (2.19) and (2.21) (with $a' = 1$ and $b' = -4$) based on the NAC $\star$-product deformation, with, in addition, an extra term

$$-g^2\, \det C\, \mathrm{Tr}\big(F\,\overline{\varphi}\big)^2\,. \tag{4.17}$$

This is, however, exactly of the form (2.23) (with $c' = -1$). As we remarked in section 2.2, such a term can be induced by a NAC $\star$-product, provided the auxiliary field $\overline{F}$ is shifted according to (2.24), and is produced at the 1-loop level.

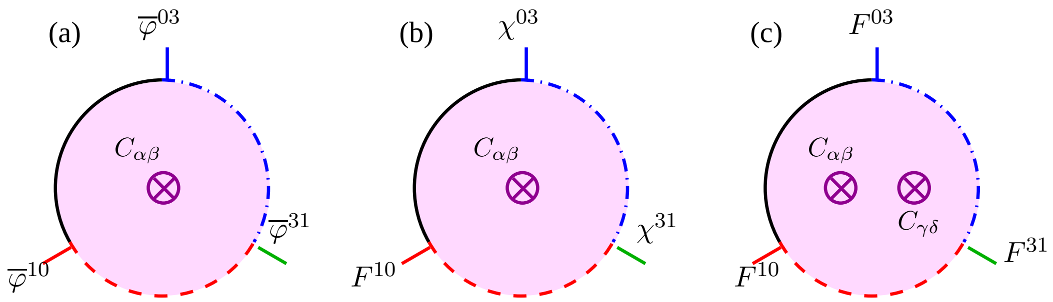

Figure 4: Examples of diagrams with R-R insertions on disks with three distinct types of boundaries that contribute to the C-deformed superpotential.

Finally, it is worth pointing out that if we insert the deformed field equation (4.11) into the linearized supersymmetry transformation (3.27) for the gaugino λ^I, we can recover exactly all non-linear terms of $\delta\lambda^I$ appearing in (2.22), including the C-dependent ones.

4.2 The superpotential sector

Let us now analyze the effects produced by the insertion of the R-R graviphoton vertex (4.1) in diagrams with three types of boundary conditions that contribute to the effective superpotential of the quiver theory. One specific example is represented in figure 4a which describes the following amplitude $\langle\!\langle\, V_{\overline{\varphi}^{01}} V_{\overline{\varphi}^{13}} V_{\overline{\varphi}^{30}} V_C \,\rangle\!\rangle$.

In order to saturate the superghost charge, one of the three vertices for the scalars can be put in the (-1)-picture and the other two in the 0-picture. Computing the corresponding string amplitude, in the field theory limit we obtain the following contribution to the lagrangian

$$2g\,\mathrm{Tr}\Big(C^{\mu\nu}\,\overline{\varphi}^{01}\partial_\mu\overline{\varphi}^{13}\partial_\nu\overline{\varphi}^{30}\Big)\,. \tag{4.18}$$

As in previous cases, also here we should consider the diagram in which the 0-picture vertices for $\overline{\varphi}$ are replaced by the vertices for the auxiliary fields $\overline{H}_\mu$, so that in the end the ordinary derivatives in (4.18) are promoted to the full covariant derivatives. Repeating this calculation for all triples of boundary conditions that can be consistently found in the quiver diagram, we finally obtain

$$2g\sum_{I\neq J\neq K}\mathrm{Tr}\Big(C^{\mu\nu}\,\overline{\varphi}^{IJ}D_\mu\overline{\varphi}^{JK}D_\nu\overline{\varphi}^{KI}\Big) \tag{4.19}$$

i.e. precisely one of the C-dependent terms of the anti-holomorphic deformed superpotential (see eq. (2.33)).

Let us now consider the diagram represented in figure 4b, which corresponds to the amplitude $\langle\!\langle\, V_{F^{01}} V_{\chi^{13}} V_{\chi^{30}} V_C \,\rangle\!\rangle$ involving fermionic vertices from the R sector. The evaluation of this amplitude is strictly analogous to what we have already described in the previous subsection and, after generalizing to all triples of consistent boundary conditions, we find

$$\frac{g}{4}\sum_{I\neq J\neq K}\mathrm{Tr}\Big(C^{\mu\nu}\,F^{IJ}\chi^{JK}\sigma^{\mu\nu}\chi^{KI}\Big) \tag{4.20}$$

which is exactly one of the terms expected from the NAC $\star$-product deformation (see eq. (2.32)).

JHEP05(2005)047

Finally, let us analyze the diagram of figure 4*c*, which, differently from all other diagrams considered so far, has two R-R insertions. It corresponds to the amplitude $\langle\!\langle\, V_{F^{01}} V_{F^{13}} V_{F^{30}} V_C\, V_C \,\rangle\!\rangle$ which is easily seen to respect all requirements in order to be non-vanishing and survive in the field theory limit. From the open string point of view, this is a 3-point amplitude which cannot be further reduced by means of suitable auxiliary fields, and thus it has to be evaluated explicitly. Since there are three open and two closed string insertions, the calculation is more involved than the ones encountered before, but it is still doable. More precisely, we have

$$\begin{aligned}\langle\!\langle\, V_{F^{01}} V_{F^{13}} V_{F^{30}} V_C\, V_C \,\rangle\!\rangle \equiv \frac{1}{2}\, C_{\rm disk} \int \frac{\prod_i dy_i\, dzd\overline{z}\, dwd\overline{w}}{dV_{\rm CKG}} \times \\ \times \langle V_{F^{01}}(p_1;y_1)\, V_{F^{13}}(p_2;y_2)\, V_{F^{30}}(p_3;y_3)\, V_C(z,\overline{z})\, V_C(w,\overline{w})\rangle\end{aligned} \tag{4.21}$$

where the symmetry factor of 1/2 accounts for the presence of two alike R-R vertices. Inserting the explicit expressions for the various ingredients and computing the world-sheet correlators, the above amplitude becomes

$$\begin{aligned}&\frac{g}{8\pi^4}\,{\rm Tr}\Big(C^{\alpha\beta}C^{\gamma\delta}F^{01}(p_1)F^{13}(p_2)F^{30}(p_3)\Big)\times\\ &\qquad\times\int\frac{\prod_i dy_i\, dzd\overline{z}\, dwd\overline{w}}{dV_{\rm CKG}}\times\\ &\times\Big\{\langle {\rm e}^{-\frac{1}{2}\phi(z)}\,{\rm e}^{-\frac{1}{2}\phi(\overline{z})}\,{\rm e}^{-\frac{1}{2}\phi(w)}\,{\rm e}^{-\frac{1}{2}\phi(\overline{w})}\rangle\times\\ &\qquad\times\langle S_\alpha(z)\,S_\beta(\overline{z})\,S_\gamma(w)\,S_\delta(\overline{w})\rangle\times\\ &\quad\times\langle\Psi^2\Psi^3(y_1)\,\Psi^3\Psi^1(y_2)\,\Psi^1\Psi^2(y_3)\times\\ &S^{---}(z)\,S^{---}(\overline{z})\,S^{---}(w)\,S^{---}(\overline{w})\rangle\\ &\qquad\langle {\rm e}^{{\rm i}p_1\cdot X(y_1)}\,{\rm e}^{{\rm i}p_2\cdot X(y_2)}\,{\rm e}^{{\rm i}p_3\cdot X(y_3)}\rangle\Big\} = \frac{g}{4\pi^4}\,{\rm Tr}\Big(\det C\,F^{01}(p_1)F^{13}(p_2)F^{30}(p_3)\Big)\times\\ &\qquad\int\frac{\prod_i dy_i\, dzd\overline{z}\, dwd\overline{w}}{dV_{\rm CKG}}\times\\ &\qquad\times\frac{(y_1-y_2)(y_1-y_3)(y_2-y_3)(z-\overline{z})(w-\overline{w})}{\prod_i(y_i-z)(y_i-\overline{z})(y_i-w)(y_i-\overline{w})}\end{aligned} \tag{4.22}$$

where in the last step we have understood, as usual, the δ-function of momentum conservation. We now exploit the $\mathrm{Sl}(2,\mathbb{R})$ invariance to fix $y_1\to\infty$, $z\to{\rm i}$ and $\overline{z}\to-{\rm i}$, so that the integrals in (4.22) become

$$4\int_{-\infty}^{+\infty}dy_2\int_{-\infty}^{y_2}dy_3\frac{(y_2-y_3)}{(y_2^2+1)(y_3^2+1)}\int_{{\rm Im}w\geq 0}dwd\overline{w}\,\frac{w-\overline{w}}{(y_2-w)(y_2-\overline{w})(y_3-w)(y_3-\overline{w})}\,. \tag{4.23}$$

Using the result (A.25) of appendix A.3, eq. (4.23) reduces to the integrals (4.8) and yields, as final result, just a factor of $4\pi^4$. Thus, the amplitude (4.22) gives rise to the term $-g\,{\rm Tr}\big(\det C\,F^{01}F^{13}F^{30}\big)$ in the effective lagrangian, which easily generalizes to

$$-\frac{g}{3}\sum_{I\neq J\neq K}{\rm Tr}\Big(\det C\,F^{IJ}F^{JK}F^{KI}\Big) \tag{4.24}$$

i.e. the last term of (2.32).

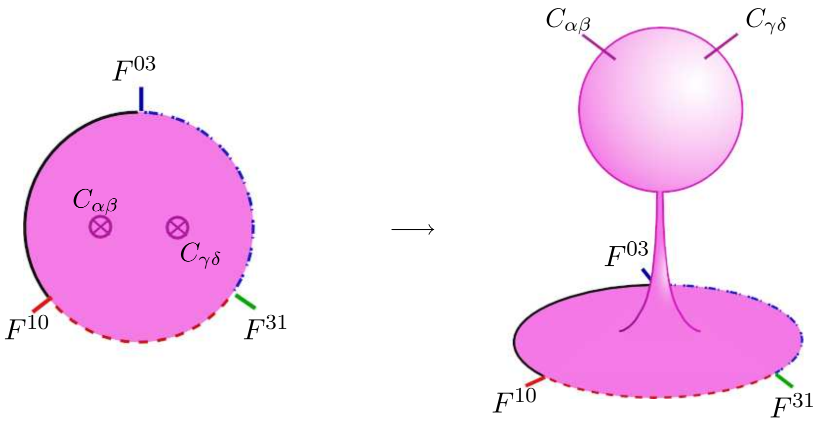

Figure 5: Factorization of the amplitude $\langle\!\langle\, V_{F^{01}} V_{F^{13}} V_{F^{30}} V_C\, V_C \,\rangle\!\rangle$ in the closed channel.

We conclude our analysis with a few general comments. All amplitudes we have computed in the presence of the R-R background involve the evaluation of some integrals over the world-sheet variables even after fixing the $\mathrm{SL}(2,\mathbb{R})$ invariance, since they contain correlation functions among more than three vertex operators. Therefore, before ascribing the final result of these amplitudes to the effective field theory action, one should study their factorization properties in order to distinguish among possible exchange contributions and select the irreducible ones. In the case of amplitudes with just a single insertion of the R-R graviphoton vertex (4.1), it is quite easy to realize that these amplitudes could be factorized only in an open string channel. However, the intermediate states which would be exchanged in such a channel are massive, since no coupling among massless states could give rise to these exchange diagrams. Amplitudes which can be factorized only on massive modes do not correspond to exchange diagrams in the effective field theory, but rather to contact interactions. This is precisely the case of the various amplitudes with a single closed string insertion that we discussed in sections 4.1 and 4.2. Things could be different, however, for the last amplitude (4.21) which has two R-R insertions, and hence can be factorized also in a closed string channel as indicated in figure 5.

However, taking into account the explicit form of the R-R vertices (4.1), it is quite easy to realize that also in this case the exchanged closed string state is massive. In fact it corresponds to the following NS-NS vertex operator

$$V(z,\overline{z}) \sim \mathrm{e}^{\mathrm{i}(\varphi_1+\varphi_2+\varphi_3)(z)}\, \mathrm{e}^{-\phi(z)}\; \mathrm{e}^{\mathrm{i}(\tilde{\varphi}_1+\tilde{\varphi}_2+\tilde{\varphi}_3)(\overline{z})}\, \mathrm{e}^{-\tilde{\phi}(\overline{z})}\, \mathrm{e}^{\mathrm{i}p\cdot X(z,\overline{z})}\,, \tag{4.25}$$

which is physical when $p^2 = -8$. Notice that in our diagram, the momentum flowing in the intermediate channel is zero, since the two external R-R vertices have both $p = 0$, and thus in the propagator of the virtual intermediate state only the mass term contributes. Again, this is not an exchange diagram of the effective theory, but rather a contact interaction, and thus the complete string amplitude (4.21) in the limit $\alpha' \to 0$ must be assigned to the effective lagrangian, as we did. Notice that this is also consistent with the fact that in our calculation we did not encounter any divergence, which, in presence of external states at zero momentum, would be typically associated to the exchange of some virtual massless particles.

In conclusion we may say that our results prove that the NAC deformation of gauge theories is completely explained by the presence of a R-R graviphoton background with constant (self-dual) field strength; this closed string background modifies the open string dynamics by introducing new types of interactions that can be easily obtained by computing mixed open/closed string amplitudes on disks with mixed boundary conditions. In this paper we have explicitly considered the specific example of the quiver theory corresponding to the orbifold $\mathbb{C}^3/(\mathbb{Z}_2\times\mathbb{Z}_2)$, but since our method is completely general, it could be applied to other orbifolds as well, or to other configurations of D-branes, like for example D-branes at angles. Furthermore, this approach can be used to analyze the effects produced by other types of closed string fluxes on the effective dynamics of open strings.

Acknowledgments

We thank Silvia Penati for several fruitful discussions.

A. Useful formulae

In this appendix we collect our conventions and several technical details that are useful for the calculations reported in the main text.

A.1 Conventions

The matrices $(\sigma^\mu)_{\alpha\dot\beta}$ and $(\bar\sigma^\mu)^{\dot\alpha\beta}$ are defined by

$$\sigma^\mu = (\mathrm{i}\vec\tau, \mathbf{1})\,, \qquad \bar\sigma^\mu = \sigma^\dagger_\mu = (-\mathrm{i}\vec\tau, \mathbf{1})\,, \tag{A.1}$$

where $\vec\tau$ are the ordinary Pauli matrices. They satisfy the Clifford algebra

$$\sigma_\mu\bar\sigma_\nu + \sigma_\nu\bar\sigma_\mu = 2\delta_{\mu\nu}\,\mathbf{1}\,, \tag{A.2}$$

and correspond to a Weyl representation of the γ-matrices acting on chiral or anti-chiral spinors ψ_α or $\psi^{\dot\alpha}$. Out of these matrices, the SO(4) generators are defined by

$$\sigma_{\mu\nu} = \frac{1}{2}(\sigma_\mu\bar\sigma_\nu - \sigma_\nu\bar\sigma_\mu)\,, \qquad \bar\sigma_{\mu\nu} = \frac{1}{2}(\bar\sigma_\mu\sigma_\nu - \bar\sigma_\nu\sigma_\mu)\,. \tag{A.3}$$

The matrices $\sigma_{\mu\nu}$ are self-dual and thus generate the $\mathrm{SU(2)_L}$ factor of SO(4); the anti self-dual matrices $\bar\sigma_{\mu\nu}$ generate instead the $\mathrm{SU(2)_R}$ factor. We raise and lower spinor indices as follows

$$\psi^\alpha = \varepsilon^{\alpha\beta}\,\psi_\beta\,, \qquad \psi_{\dot\alpha} = \varepsilon_{\dot\alpha\dot\beta}\,\psi^{\dot\beta} \tag{A.4}$$

where $\varepsilon^{12} = \varepsilon_{12} = -\varepsilon^{\dot1\dot2} = -\varepsilon_{\dot1\dot2} = +1$. From these rules it follows that

$$\psi^\alpha\,\psi^\beta = -\frac{1}{2}\,\varepsilon^{\alpha\beta}\,\psi\,\psi\,, \qquad \bar\psi^{\dot\alpha}\,\bar\psi^{\dot\beta} = -\frac{1}{2}\,\varepsilon^{\dot\alpha\dot\beta}\,\bar\psi\,\bar\psi \tag{A.5}$$

and

$$\psi\,\sigma^\mu\bar\psi\;\,\psi\,\sigma^\nu\bar\psi = \frac{1}{2}\,\psi\psi\;\bar\psi\bar\psi\;\delta^{\mu\nu}\,. \tag{A.6}$$

A.2 World-sheet correlation functions

We report here some correlation functions among conformal fields that are needed for the calculation of the string amplitudes of sections 3 and 4.

Space-time correlators:

$$\langle :\psi^\mu\psi^\nu:(y)\,S_\alpha(z)\,S_\beta(\overline{z})\rangle = \frac{1}{2}(\sigma^{\mu\nu})_{\alpha\beta}(z-\overline{z})^{\frac{1}{2}}\,(y-z)^{-1}\,(y-\overline{z})^{-1}\,. \tag{A.7}$$

$$\langle \psi^\mu(y_1)\,\psi^\nu(y_2)\,S_\alpha(z)\,S_\beta(\overline{z})\rangle = A(y_1,y_2,z,\overline{z})\,\delta^{\mu\nu}\,\varepsilon_{\alpha\beta}+B(y_1,y_2,z,\overline{z})(\sigma^{\mu\nu})_{\alpha\beta} \tag{A.8}$$

where

$$A(y_1,y_2,z,\overline{z}) = \frac{1}{2}\,\frac{(y_1-z)(y_2-\overline{z})+(y_2-z)(y_1-\overline{z})}{(y_1-y_2)\big[(y_1-z)(y_1-\overline{z})(y_2-z)(y_2-\overline{z})(z-\overline{z})\big]^{\frac{1}{2}}} \tag{A.9}$$

and

$$B(y_1,y_2,z,\overline{z}) = -\frac{1}{2}\left[\frac{(z-\overline{z})}{(y_1-z)(y_1-\overline{z})(y_2-z)(y_2-\overline{z})}\right]^{\frac{1}{2}}. \tag{A.10}$$

$$\begin{aligned}\langle S_\gamma(y_2)\,S_\delta(y_3)S_\alpha(z)\,S_\beta(\overline{z})\rangle &= \big[\varepsilon_{\gamma\delta}\,\varepsilon_{\alpha\beta}\,(y_2-\overline{z})(y_3-z)-\varepsilon_{\gamma\beta}\,\varepsilon_{\delta\alpha}\,(y_2-y_3)(z-\overline{z})\big]\times \\ &\quad\times\big[(y_2-y_3)(y_2-z)(y_2-\overline{z})(y_3-z)(y_3-\overline{z})(z-\overline{z})\big]^{-\frac{1}{2}}\,.\end{aligned} \tag{A.11}$$

Internal space correlators:

$$\langle \Psi^1(y_1)\,\Psi^2\Psi^3(y_2)\,S^{---}(z)\,S^{---}(\overline{z})\rangle = |y_1-z|^{-1}\,|y_2-z|^{-2}\,(z-\overline{z})^{\frac{3}{4}}\,. \tag{A.12}$$

$$\begin{aligned}&\langle \Psi^2\Psi^3(y_1)\,\Psi^3\Psi^1(y_2)\,\Psi^1\Psi^2(y_3)\,\times \\ &\times S^{---}(z_1)\,S^{---}(\overline{z}_1)\,S^{---}(z_2)\,S^{---}(\overline{z}_2)\rangle = \prod_{i=1}^{3}\prod_{a=1}^{2}|y_i-z_a|^{-2}\prod_{i<j}(y_i-y_j)\times \\ &\qquad\times\prod_{a<b}|z_a-z_b|^{\frac{3}{2}}\prod_{a,b}|z_a-\overline{z}_b|^{\frac{3}{2}}\,.\end{aligned} \tag{A.13}$$

Superghost correlators:

$$\langle \mathrm{e}^{-\phi(y_1)}\,\mathrm{e}^{-\frac{1}{2}\phi(y_2)}\,\mathrm{e}^{-\frac{1}{2}\phi(y_3)}\rangle = (y_1-y_2)^{-\frac{1}{2}}\,(y_1-y_3)^{-\frac{1}{2}}\,(y_2-y_3)^{-\frac{1}{4}}\,, \tag{A.14}$$

$$\begin{aligned}\langle \mathrm{e}^{-\frac{1}{2}\phi(y_1)}\,\mathrm{e}^{-\frac{1}{2}\phi(y_2)}\,\mathrm{e}^{-\frac{1}{2}\phi(y_3)}\,\mathrm{e}^{-\frac{1}{2}\phi(y_4)}\rangle &= \Big[(y_1-y_2)\,(y_1-y_3)\,(y_1-y_4)\,(y_2-y_3)\,\times \\ &\quad\times(y_2-y_4)\,(y_3-y_4)\Big]^{-\frac{1}{4}}\,.\end{aligned} \tag{A.15}$$

A.3 A useful integral

In the calculation of the string amplitude $\langle\!\langle\, V_{F^{01}}V_{F^{13}}V_{F^{30}}V_C\,V_C\,\rangle\!\rangle$, one of the ingredients is the following integral in the upper half complex plane $\mathcal{H}_+$

$$I(a,b) = \int_{\mathcal{H}_+} dw\,d\overline{w}\frac{(w-\overline{w})}{(w-a)(\overline{w}-a)(w-b)(\overline{w}-b)} \tag{A.16}$$

with $(a, b \in \mathbb{R}, a > b)$ (see eq. (4.23)). As it stands, the integral $I(a, b)$ is formally divergent. We may regularize it by excluding the real axis from the integration region, i.e. we take $\operatorname{Im} w \geq \epsilon$, where ϵ will be sent to zero at the end of the calculation. Notice that this regularization prescription is precisely what is required in mixed open/closed string amplitudes. With this in mind, after applying Stoke's theorem, we have

$$I(a, b; \epsilon) = \oint_{\partial\mathcal{H}_{+\epsilon}} d\overline{w} \frac{\ln\left[(w-a)(w-b)\right]}{2(\overline{w}-a)(\overline{w}-b)} + \oint_{\partial\mathcal{H}_{+\epsilon}} dw \frac{\ln\left[(\overline{w}-a)(\overline{w}-b)\right]}{2(w-a)(w-b)} . \tag{A.17}$$

On the integration path, we have

$$w = x + \mathrm{i}\epsilon\,, \qquad \overline{w} = x - \mathrm{i}\epsilon\,, \qquad dw = d\overline{w} = dx \tag{A.18}$$

with $-\infty < x < +\infty$. Thus, the first integral in (A.17) becomes

$$I_1(a, b; \epsilon) = \int_{-\infty}^{+\infty} dx \frac{\ln\left[(x-a+\mathrm{i}\epsilon)(x-b+\mathrm{i}\epsilon)\right]}{2(x-a-\mathrm{i}\epsilon)(x-b-\mathrm{i}\epsilon)} \tag{A.19}$$

which can be easily evaluated using Jordan's lemma and residues theorem. In fact, we get

$$I_1(a, b; \epsilon) = 2\pi\mathrm{i}\left\{\frac{\ln\left[(2\mathrm{i}\epsilon)(a-b+2\mathrm{i}\epsilon)\right]}{2(a-b)} + \frac{\ln\left[(b-a+2\mathrm{i}\epsilon)(2\mathrm{i}\epsilon)\right]}{2(b-a)}\right\} \tag{A.20}$$

$$= \frac{\pi\mathrm{i}}{a-b}\Big[\ln(a-b+2\mathrm{i}\epsilon) - \ln(b-a+2\mathrm{i}\epsilon)\Big] . \tag{A.21}$$

Now we can safely take the limit $\epsilon \to 0^+$, and using the fact that

$$\lim_{\epsilon \to 0^+}\Big[\ln(a-b+2\mathrm{i}\epsilon) - \ln(b-a+2\mathrm{i}\epsilon)\Big] = \pi\mathrm{i}\,, \tag{A.22}$$

we finally get

$$I_1(a, b) = \frac{\pi^2}{a-b} . \tag{A.23}$$

The calculation of the second integral in (A.17) proceeds along the same lines and yields the same result,

$$I_2(a, b) = \frac{\pi^2}{a-b} ; \tag{A.24}$$

thus in the end we have

$$I(a, b) = I_1(a, b) + I_2(a, b) = \frac{2\pi^2}{a-b} . \tag{A.25}$$

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